Adaptive model reduction and feedback control of transport reaction processes

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***Abstract*— The problem of robust feedback control of spatially distributed processes described by dissipative partial differential equations (PDEs) is considered. Typically, this problem is addressed through model reduction where finite dimensional approximations to the original PDE system are derived. A common approach to this task is the Karhunen-Loève expansion combined with the method of snapshots. To circumvent the issue of *a priori* availability of a sufficiently large ensemble of PDE solution data, we focus on the recursive computation of eigenfunctions as additional data from the process become available. Initially, an ensemble of eigenfunctions is constructed with a relatively small number of snapshots. The dominant eigenspace of this ensemble is then identified to compute the empirical eigenfunctions required for model reduction. This dominant eigenspace is reevaluated with the addition of new snapshots the dominant eigenspace is reevaluated and its dimensionality may increase or decrease. Because this dimensionality is typically small the computational burden is also small. This approach is applied to representative example of a thin catalytic rod, where zero-th order exothermic reaction is taking place. The available model for heat transfer in this catalytic rod was assumed to contain uncertain parameters (heat of reaction) and as a result standard control design techniques cannot be used. We demonstrate the effectiveness of the proposed methodology in this case by designing robust controllers to stabilize an unsteady process operating condition in the thin-catalytic rod.**

I. INTRODUCTION

Most of the processes relevant to the chemical process industry necessitate the consideration of transport phenomena (fluid flow, heat and mass transfer) often coupled with chemical reactions. Examples range from reactive distillation in petroleum processing to plasma enhanced chemical vapor deposition, etching and metallorganic vapor phase epitaxy in semiconductor manufacturing. Mathematical descriptions of these transport-reaction processes can be derived from dynamic conservation equations and usually involve highly dissipative (typically parabolic) partial differential equation (PDEs) systems. The problem of feedback control of such processes is nontrivial owing to these spatially distributed mathematical descriptions. Some of the approaches used to compute feedback control include standard Galerkin's method [4], [3], [5], approximate inertial manifolds[6], Lyapunov function techniques [2], [1]. However, the above approaches for model reduction cannot be directly applied to systems which have nonlinear spatial differential operators or to problems defined over irregular spatial domains, since the associated eigenvalue-eigenvector problem then cannot be analytically solved, and the proposed methods to expand the solution of the PDE systems then loose their computational advantages. To overcome this limitation, researchers have focussed on data-driven methods such as Proper-orthogonal decomposition [13], [7], [12], [11], [15].

The above data-driven methods though assume an *a priori* availability of a large ensemble of snapshots to characterize the behavior of the process and identify new trends during the course of process evolution using the basis functions computed from the *a priori* available snapshot set. However, generating such an ensemble is a very involved task as it necessitates using a suitably designed input [7], [14] to excite all the modes and gather all the process information beforehand; this is computationally difficult and experimentally infeasible. This difficult task is especially critical when control of the process is considered since when new trends in the data appear they need to be accounted for and usually efficiently suppressed by a successful controller [8], [9].

To circumvent the latter problems, an algorithm that allows for recursive update of empirical eigenfunctions *once new measurements from the process become available* was proposed in [15], [10] (which we title adaptive proper orthogonal decomposition). The approach is based on the computation of an approximation of the eigenspace of the covariance matrix corresponding to its significant eigenvalues. The dominant eigenspace is updated recursively as new snapshots from the process are added to the ensemble, simultaneously increasing or decreasing its dimensionality if required. In the present work, this adaptive model reduction methodology is implemented on a process with time varying uncertain parameters. A robust controller is designed to achieve the desired uncertainty attenuation in the closed-loop system. We evaluate the effectiveness of the proposed approach numerically through a representative example of a diffusion-reaction process (heat transfer on a thin catalytic rod where zero-th order exothermic reaction takes place).

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